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SOUND TRANSMISSION THROUGH LAYERED MEDIA: A SIMPLIFIED METHOD O--ETC(U)

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SOUND TRANSMISSION THROUGH LAYERED MEDIA: A SIMPLIFIED METHOD OF ANALYSIS

by

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ABSTRACT

The four-pole method common to electrical circuit analysis is applied to analysis of sound transmission through layered media. This method provides a characterization of the system by means of a 2 x 2 matrix, called the four-pole matrix of the system. A general four-pole matrix is derived for a single viscoelastic layer. A sound transmission system consisting of many layers is analyzed by specializing this matrix for each layer, and multiplying the resulting matrices, giving an equivalent four-pole matrix for the system. Electrical analogs are easily computed from the terms of the four-pole matrices.

BRIEF DISCUSSION OF THE FOUR-POLE PARAMETER METHOD

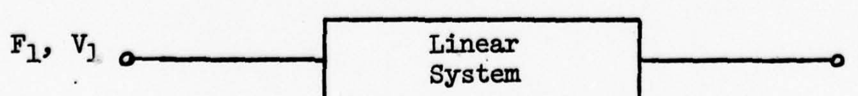

Figure 1

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The problem considered is the determination for a linear mechanical system represented in Figure 1 of the input force and velocity (F_1, V_1) in terms of the output force and velocity (F_2, V_2), and quantities that depend on the character of the system. If steady-state harmonic inputs and outputs only are considered, the relationships

$$F_1 = \alpha_{11} F_2 + \alpha_{12} V_2$$

$$V_1 = \alpha_{21} F_2 + \alpha_{22} V_2$$

are valid(1), where the α_{ij} depend on the system under consideration. The α_{ij} are called the four-pole parameters of the system. Determination of the α_{ij} is a relatively simple matter. It is merely assumed that the input and output have representations* $A \ e^{i\omega t}$ where A., ω are constants (A is the complex amplitude and ω the angular frequency), t is time, i is $\sqrt{-1}$. The equations for velocity and force are written by Newton's laws and constitutive relations of the system, and evaluated at input and output terminals of the system, and the α_{ij} are solved for algebraically. A simple example of a spring - mass system is presented to illustrate.

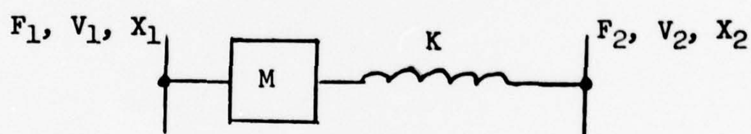


Figure 2

For the system represented in figure 2., Newton's second law yields:

$$F_1 - F_2 = M \frac{dV}{dt} \qquad (1)$$

The constitutive relation for the spring is:

$$F_2 = K(X_1 - X_2) \qquad (2)$$

* For an explanation of this representation, see reference (2) sect. 3-3.

It is now assumed that the input and output forces and velocities are sinusoidal with constant amplitude, i.e., $F_j = A_j\, e^{i\omega t}$, $V_j = B_j C^{i\omega t}$, $j = 1,2$. Then from (1) and (2):

$$F_1 - F_2 = i\omega m V_1$$

$$i\omega F_2 = K(V_1 - V_2)$$

Solving for F_1, V_1 :

$$F_1 = (1 - \frac{\omega^2 m}{K})\, F_2 + i\omega m\, V_2 \qquad (3)$$

$$V_1 = \frac{i\omega}{K} F_2 + V_2 \qquad (4)$$

Using relations (3) and (4), or their inverse:

$$\begin{Bmatrix} F_2 \\ V_2 \end{Bmatrix} = (\beta\, ij) \begin{Bmatrix} F_1 \\ V_1 \end{Bmatrix}, \quad (\beta\, ij) = (\alpha ij)^{-1}$$

any two of the four quantities may be computed if the other two are known. For example, if the system is driven as shown in figure 3, then $V_2 = 0$, $F_1 = A\, e^{i\omega t}$. Then from (3) and (4):

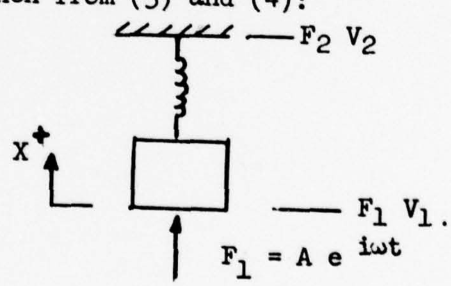


Figure 3

$$Ae^{i\omega t} = (1 - \frac{\omega^2 m}{K})\ F_2 \qquad F_2 = \left[\frac{A}{1 - \frac{\omega^2 m}{K}}\right] e^{i\omega t}$$

$$V_1 = \frac{i\omega}{K} F_2 = \frac{i\omega}{K} \left[\frac{A}{1 - \frac{\omega^2 m}{K}}\right] e^{i\omega t}$$

Referring to Figure 4, the force voltage analogy* can be obtained from the four-pole parameters by:

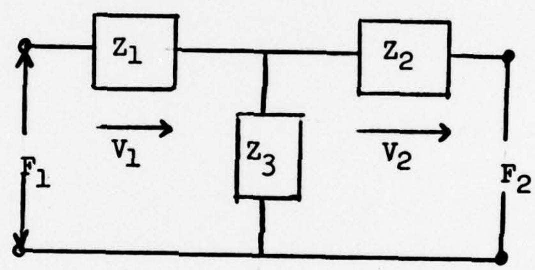


Figure 4

the use of the impedance formulae:

$$Z_1 = \frac{\alpha 11 - 1}{\alpha 21} \tag{5}$$

$$Z_2 = \frac{\alpha 22 - 1}{21} \tag{6}$$

$$Z_3 = \frac{1}{\alpha 21} \tag{7}$$

The analogous quantities referred to Figure 4 are given in Table 1.

*Among others reference (3) explains mechanical electrical analogies.

Mechanical	Electrical
Input force F_1	Input voltage F_1
Output force F_2	Output voltage F_2
Input velocity V_1	Current V_1
Output velocity V_2	Current V_2

Table 1

For the above problem:

$$Z_1 = \frac{\omega^2 m}{k} \Big/ \frac{i\omega}{k} = i\omega m$$

$$Z_2 = 0$$

$$Z_3 = \frac{k}{i\omega} = \frac{1}{i\omega(\frac{1}{k})}$$

If it is specified that $V_2 = 0$, $F_1 = A\,e^{i\omega t}$, the force-voltage analogous circuit for the system shown in Figure 3 is as shown in Figure 5.

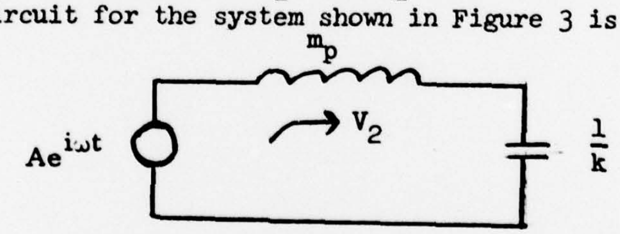
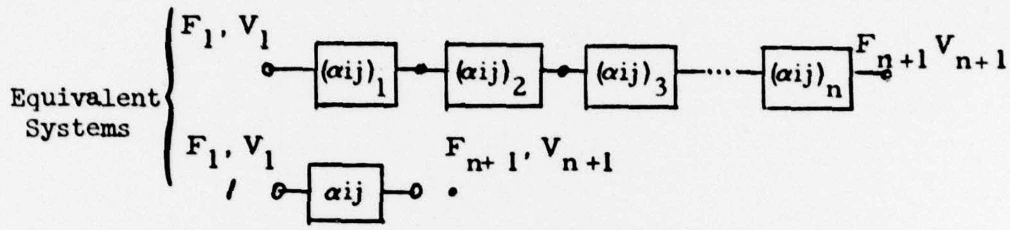


Figure 5

For n systems connected in tandem, the k^{th} system from the input terminal being characterized by the four-pole matrix $(\alpha ij)_k$, the equivalent single four-pole matric (αij) for the entire system is[1]:

$$\alpha ij = \prod_{k=1}^{m} (\alpha ij)_k \tag{8}$$

This situation is illustrated in Figure 6:



$$\begin{Bmatrix} F_1 \\ V_1 \end{Bmatrix} = \left[\prod_{k=1}^{n} (\alpha ij)_k \right] \begin{Bmatrix} F_{n+1} \\ V_{n+1} \end{Bmatrix}$$

Figure 6

It is noted that the order of multiplication of the four-pole matrices in formula (8) is required to be the same as the order of connection of the subsystems, going from input to output.

DERIVATION OF THE FOUR-POLE PARAMETERS FOR A GENERAL VISCOELASTIC LAYER

Consider a layer of viscoelastic material acted upon by sound pressure fields as shown in Figure 7, where

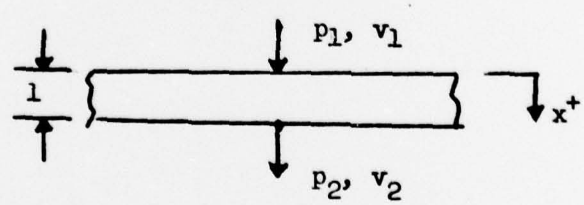


Figure 7

p represents pressure, and v represents velocity. Using a four-pole technique similar to that explained in the previous section, we can compute a four-pole matrix (αij) such that:

$$\begin{Bmatrix} p_1 \\ v_1 \end{Bmatrix} = (\alpha ij) \begin{Bmatrix} p_2 \\ v_2 \end{Bmatrix}$$

Furthermore, an electrical analog for the layer can be computed using formulae (5), (6), and (7). Electrical analogs are commonly used in acoustical problems, and can be obtained by other means; see for example reference (4). The most important advantage of using a four-pole technique accrues when a plate is composed of layers, which may have different viscoelastic properties. The equivalent four-pole (and subsequently the electrical analog) for the entire composite is obtained rather simply by application of formula (8). The four-pole parameters are computed below for a "typical" layer, and the four-pole matrix for a specific layer is obtained by substituting the appropriate parameters into this "typical" matrix. It is emphasized that the analysis that follows is only valid for plane waves of pure harmonic character propagating through linear viscoelastic materials normal to the plane of the layer.

The equation of motion (9) and the constitutive relation (10) under the assumptions above for the layer in Figure 7 are:

$$c^2 \frac{\partial^2 u}{\partial X^2} = \frac{\partial^2 u}{\partial t^2} \tag{9}$$

$$\sigma = -M \frac{\partial u}{\partial X} \tag{10}$$

where the terms are defined below:

X = the lagrangian coordinate normal to the plane of the plate

u = displacement in the X direction

M = a complex modulus*, $M = \hat{M}(1 + i\eta)$

$\hat{M}$ = the appropriate elastic modulus of the system

$i = \sqrt{-1}$

η = a loss factor depending on the material, its temperature, and the frequency of excitation (5)

*The concept of a complex modulus is explained in reference (5). The addition of an imaginary term to the modulus allows a representation that includes internal damping effects for pure harmonic motion.

$c^2 = \frac{M}{\rho}$, if M is real, c is the velocity of sound in the medium

ρ = mass density of material

t = time

σ = complex stress in the medium

For a layer whose lateral dimensions are large compared to its thickness (5):

$$M \doteq B + \frac{4}{3} G, \text{ where}$$

B = complex bulk modulus

G = complex rigidity modulus

For (9), we assume a solution:

$$u = U_o \, c^{\,i(\omega t + kx)} \qquad (11)$$

Performing the appropriate differentiations on (11), and substituting these into (9) results in $k = \pm \frac{\omega}{c}$, so that we set:

$$u = U_- e^{i\omega(t + \frac{x}{c})} + U_+ e^{i\omega(t - \frac{x}{c})} \qquad (12)$$

Setting

$$v = \frac{\alpha u}{\alpha t} = i\omega e^{i\omega t} \left[U_- e^{\frac{i\omega x}{c}} + U_+ e^{\frac{-i\omega x}{c}} \right] , \qquad (13)$$

and using (10)

$$\sigma = -M \frac{\alpha u}{\alpha x} = -M \frac{i\omega}{c} e^{i\omega t} \left[U_- e^{\frac{i\omega x}{c}} - U_+ \, c^{\frac{-i\omega x}{c}} \right] \qquad (14)$$

Equations (13) and (14) are evaluated at the input and output sides of the plate:

$$\sigma_1 = p_1 = -M \left.\frac{\alpha u}{\alpha x}\right|_{x=0} = -M \frac{i\omega}{c} e^{i\omega t} \left[U_- - U_+\right] \tag{15}$$

$$v_1 = \left.\frac{\alpha u}{\alpha t}\right|_{x=0} = i\omega e^{i\omega t} \left[U_- + U_+\right] \tag{16}$$

$$\sigma_2 = p_2 = -M \left.\frac{\alpha u}{\alpha x}\right|_{x=l} = -M \frac{i\omega}{c} e^{i\omega t} \left[U_- e^{i\frac{\omega l}{c}} - U_+ e^{\frac{-i\omega l}{c}}\right] \tag{17}$$

$$v_2 = \left.\frac{\alpha u}{\alpha t}\right|_{x=l} = i\omega e^{i\omega t} \left[U_- e^{\frac{i\omega l}{c}} \quad U_+ e^{-i\frac{\omega l}{c}}\right] \tag{18}$$

Equations (17) and (18) can be solved simultaneously for $U_- e^{i\omega t}$ and $U_+ e^{i\omega t}$, resulting in:

$$U_- e^{i\omega t} = \frac{1}{2i\omega} e^{\frac{-i\omega l}{c}} \left[v_2 - \frac{c}{M} p_2\right] \tag{19}$$

$$U_+ e^{i\omega t} = \frac{1}{2i\omega} e^{\frac{i\omega l}{c}} \left[v_2 + \frac{c}{M} p_2\right] \tag{20}$$

Substituting (19) and (20) into (15) and (16):

$$p_1 = (\cos \frac{\omega l}{c})\, p_2 + (\frac{iM}{c} \sin \frac{\omega l}{c})\, v_2 \tag{21}$$

$$v_1 = (\frac{ic}{M} \sin \frac{\omega l}{c})\, p_2 + (\cos \frac{\omega l}{c})\, v_2 \tag{22}$$

Now:

$$\frac{M}{C} = \frac{\rho M}{\rho C} = \frac{\rho}{C}\left(\frac{M}{\rho}\right) = \frac{\rho}{C} C^2 = \rho C = R_p \tag{23}$$

Substituting (23) into (21) and (22), and writing in vector form:

$$\begin{Bmatrix} p_1 \\ v_1 \end{Bmatrix} = \begin{bmatrix} \cos\frac{\omega l}{c} & iR_p \sin\frac{\omega l}{c} \\ \frac{-1}{iR_p} \sin\frac{\omega l}{c} & \cos\frac{\omega l}{c} \end{bmatrix} \begin{Bmatrix} p_2 \\ v_2 \end{Bmatrix} \tag{24}$$

Equation (24) is a "typical" four pole representation for the system of Figure 7. It can be used for any layer in a composite system provided that M, l, and ρ for the material of the layer are substituted into the four-pole matrix. A typical electrically analogous T-network is obtained below by using equations (5), (6), and (7), and is shown in Figure 8.

$$Z_1 = \frac{\alpha_{11}-1}{\alpha_{21}} = \frac{\cos\frac{\omega l}{c} - 1}{\frac{-1}{iR_p}\sin\frac{\omega l}{C}} = iR_p\left(\frac{1-\cos\frac{\omega l}{c}}{\sin\frac{\omega l}{c}}\right)$$

$$= iR_p \tan\frac{\omega l}{2c}$$

$$Z_2 = \frac{\alpha_{22}-1}{\alpha_{21}} = Z_1 = i R_p \tan\frac{\omega l}{2c}$$

$$Z_3 = \frac{1}{\alpha_{21}} = \frac{-iR_p}{\sin\frac{\omega l}{c}} = -iR_p \csc\frac{\omega l}{c}$$

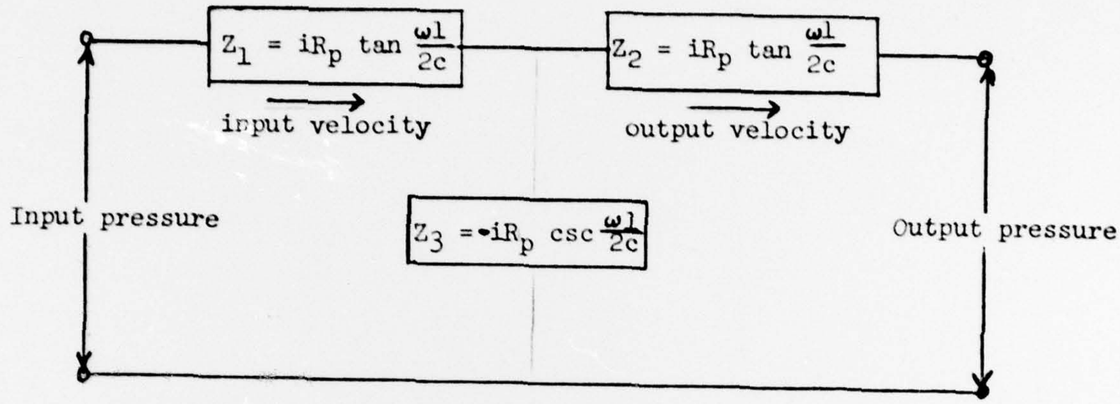


Figure 8

Cascaded plates are represented by cascading the individual T-networks as shown in Figure 9. An equivalent single T-network can be computed by using Norton's and Thevinin's theorems directly on the electrical analog or perhaps more easily by using equation (8) to obtain an equivalent single four-pole matrix, and computing a T-network from this using equations (5), (6), and (7).

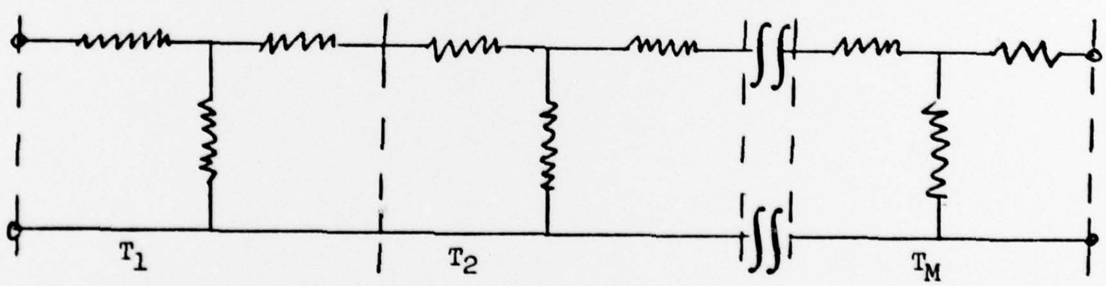


Figure 9

INPUT AND OUTPUT NETWORKS

The electrical analogy of Figure 9 is not complete until input and output networks are specified. Figure 10 represents a composite plate upon which is incident a sound pressure p_I.

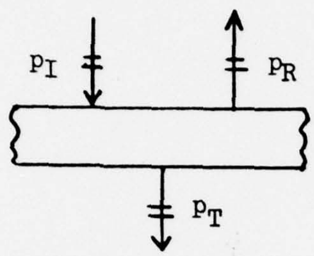


Figure 10

There is a reflected pressure p_R, and a transmitted pressure p_T. Comparing Figures 10 and 7,

$$p_1 = p_I + p_R$$

$$p_2 = p_T$$

The medium on the input side of the plate has specific acoustic impedance $R_1 = \rho_1 C_1$. The specific radiation impedance on both sides of the plate is assumed equal to the specific acoustic impedance, since the plate is of large lateral extent. The specific radiation impedance on the output side of the plate is $R_2 = \rho_2 C_2$ (6). If v_T is the velocity associated with the transmitted wave (6),

$$p_2 = \rho_2 C_2 v_2 = R_2 v_2$$

Since R_2 is real, the pressure-voltage analog of the output is as shown in Figure 11.

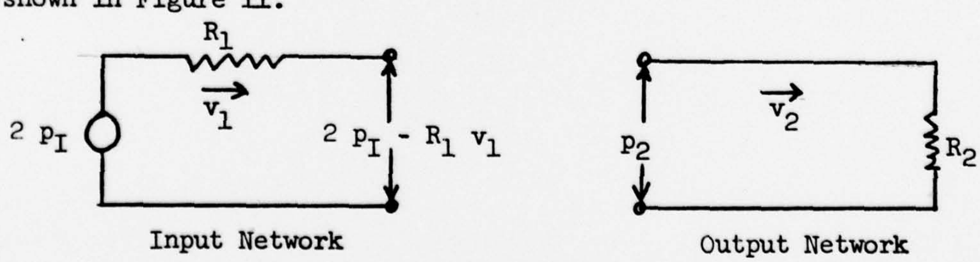


Figure 11

Similarly, denoting by v_I and v_R the velocities associated with the incident and reflected waves, respectively[6].

$$p_I = R_1 v_1$$

$$p_R = -R_1 v_R$$

Continuity at the input face of the plate requires that:

$$v_1 = v_I + v_R$$

Hence:

$$p_1 = p_1 + p_R = p_I - R_1 v_R = p_I - R_1(v_1 - v_I) = p_I - R_1 v_1 + R_1 v_I = 2 p_I - R_1 v_1$$

The input voltage to the network of Figure 9 is then $2 p_I - R_1 v_1$. This can be realized by the input network shown in Figure 11. The total network for the plate is shown in Figure 12. The analogous quantities of interest are listed in Table 2.

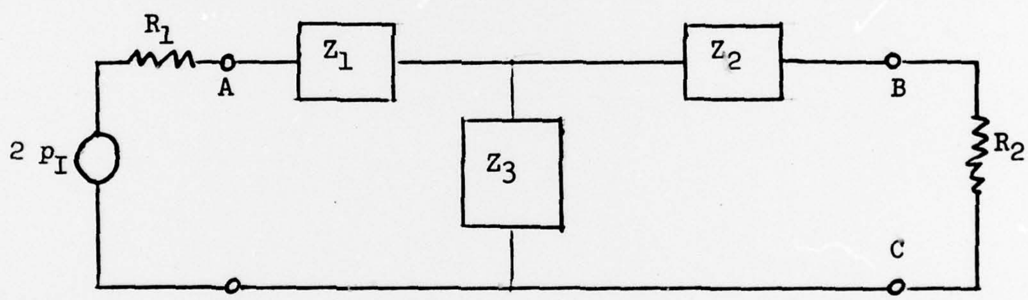


Figure 12

Acoustic	Electrical
Input Velocity v_1	Current through R_1
Transmitted Pressure p_t	Voltage Drop BC
Reflected Pressure p_t	Voltage Drop AC minus $\frac{1}{2}$ source voltage current through R_2
Output Velocity V_2	Current through R_2

Table 2

APPLICATION OF THE FOUR-POLE TECHNIQUE TO COMPUTE SOUND TRANSMISSION

Some Specific Examples

Case I - steel plate, thickness small compared to an acoustic wavelength $(\frac{\omega l}{c} < .1)$:

$$M = \hat{M} = \hat{E} + 4/3\,\hat{G}, \quad (\eta = 0)$$

Since $\frac{\omega l}{c}$ is small, we may use the approximations

$$\tan\frac{\omega l}{2c} = \frac{\omega l}{2c}, \quad \csc\frac{\omega l}{c} = \frac{c}{\omega l}$$

Referring to Figure 8,

$$Z_1 = Z_2 = i\,R_p\,\frac{\omega l}{2c} = i\rho c\,\frac{\omega l}{2c} = i\omega\,\frac{\rho l}{Z}$$

ρl is the mass per unit area of the plate, and is denoted m_p.

Then

$$Z_1 = Z_2 = i\omega\,\frac{m_p}{2}$$

$$Z_3 = -iR_p\,\frac{c}{\omega l} \quad -1\,\frac{\rho c^2}{\omega l} = \frac{-i\hat{M}}{\omega l} = \frac{1}{i\omega(\frac{1}{\hat{M}})}$$

The electrical T-network corresponding to Figure 8 is shown in Figure 13.

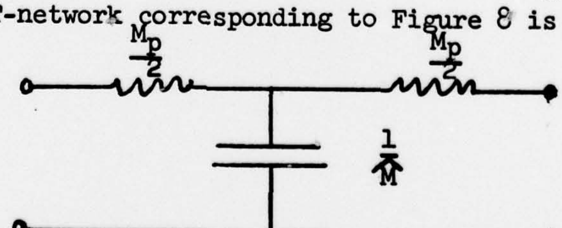


Figure 13

If the plate thickness is on the order of 1 inch, $\frac{1}{\hat{M}}$ is on the order of 10^{-7}, while m_p is on the order of 1. Hence we may disregard the capacitor of Figure 13 and the analogous circuit corresponding to Figure 11 becomes that shown in Figure 14.

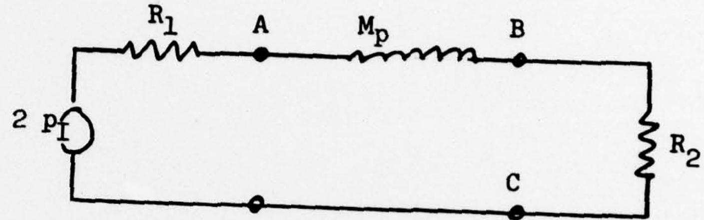


Figure 14

Two important acoustic parameters for a plate are the transmission loss (TL), and reflection loss (RL). These are computed for this case:

$$TL = 10 \log \left| \frac{p_I}{p_T} \right|^2$$

$$= 10 \log \left(\frac{(R_1 + R_2)^2 + \omega^2 m_p{}^2}{4 R_2{}^2} \right)$$

$$RL = 10 \log \left| \frac{p_I}{p_R} \right| = 10 \log \left| \frac{p_I}{\left(\frac{2 p_I}{R_1 + R_2 + i\omega m_p}\right) R_2 - p_1} \right|$$

$$= 10 \log \left(\frac{(R_1 + R_2)^2 + \omega^2 m_p{}^2}{(R_2 - R_1)^2 + \omega^2 m_p{}^2} \right)$$

If $R_1 = R_2 = R$,

$$TL \cong 10 \log \left(1 + \frac{\omega^2 m_p{}^2}{4 R^2} \right)$$

$$RL = 10 \log \left(1 + \frac{4 R^2}{\omega^2 m_p^2}\right)$$

Case II - Viscoelastic Material, $\left|\frac{\omega l}{c}\right| < .1$

$$M = E + \frac{4}{3} G = \hat{M}(1 + i\eta)$$

Using the same approximations as Case I,

$$Z_1 = Z_2 = \frac{i\omega m_p}{2}$$

$$Z_3 = \frac{-i\rho c^2}{\omega l} = \frac{-iM}{\omega l} = \frac{\hat{M}\eta}{\omega l} + \frac{1}{i\omega(\frac{l}{\hat{M}})}$$

In this case, $\frac{l}{\hat{M}}$ may not be neglected, and the analogous T-network is that of Figure 15:

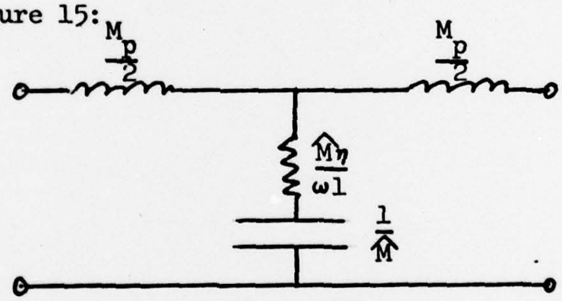


Figure 15

Cases I and II can be combined to yield an electrical analog for a steel plate with a thin viscoelastic damping coating on the incident side. This circuit is shown in Figure 16 where the damping material constants are subscripted with d, and those of the steel plate with p. The acoustic impedance of both sides of the composite is assumed to be R.

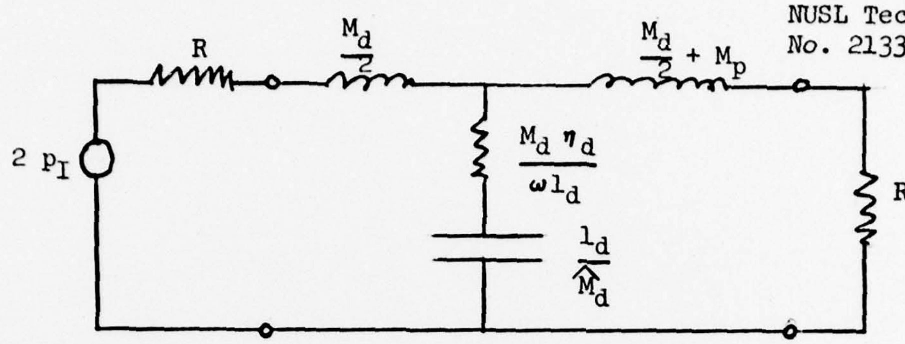


Figure 16

CONCLUSION

Transmission of sound through layered media is easily analyzed by the use of the four-pole method. The basic pole matrix (the matrix of equation (24)) is used for each layer of the composite structure, by substituting the appropriate constants into αij. Formula (8) is then used to obtain an equivalent four-pole matrix for the entire structure, and if an electrical analog is desired, equations (5), (6), and (7) are used to obtain the T-network that appears in the analogous circuit of Figure 11.

Application of this method should be useful for analysis of sonar dome construction, baffle design, isolation mountings, and other similar problems.

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